

CSSE 490

Network Security

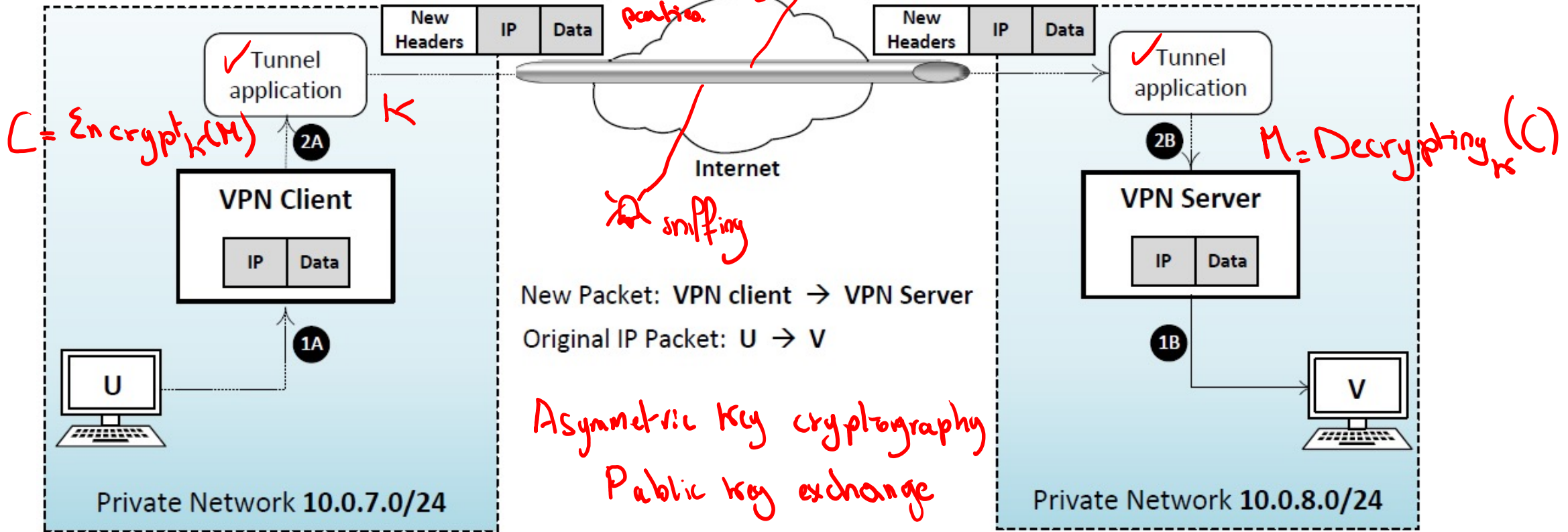
Day 30: Public-Key Encryption

Outline

- ❑ Recap: SSL/TLS-based VPN
- ❑ A bit of history
- ❑ Public-key cryptography
- ❑ Diffie-Hellman key exchange
- ❑ Rivate-Shamir-Adleman (RSA) math background
- ❑ RSA algorithm

SSL/TLS based VPN

Symmetric key encryption: Shared K known only to communicating parties. Secure channel over insecure Internet encrypt/decrypt



A bit of history



Whitfield Diffie

Martin Hellman

2015 Turing Award Winner



Clifford Cocks



Leonard Adleman



Ron Rivest



Adi Shamir

2002 Turing Award Winner

Public-key cryptography

Generate two sets of keys

- ❑ Public key for encryption
- ❑ Private key for decryption

$$C = \text{Encrypt}_{\text{publickey}}(M) \xrightarrow{\text{insecure channel}}$$
$$M = \text{Decrypting}_{\text{private key}}(C)$$

Also, for authentication

- ❑ Private key for signature
- ❑ Public key for verifying the signature

Diffie-Hellman key exchange

- ❑ Exchange secret key over insecure channel
- ❑ Communicating parties agree on:
 - Number p : big prime number (2048-bit number)
 - Generator g : small prime number
- ❑ Alice picks x , a random positive integer, $x < p$
- ❑ Bob pick y , a random positive integer, $y < p$

Public information

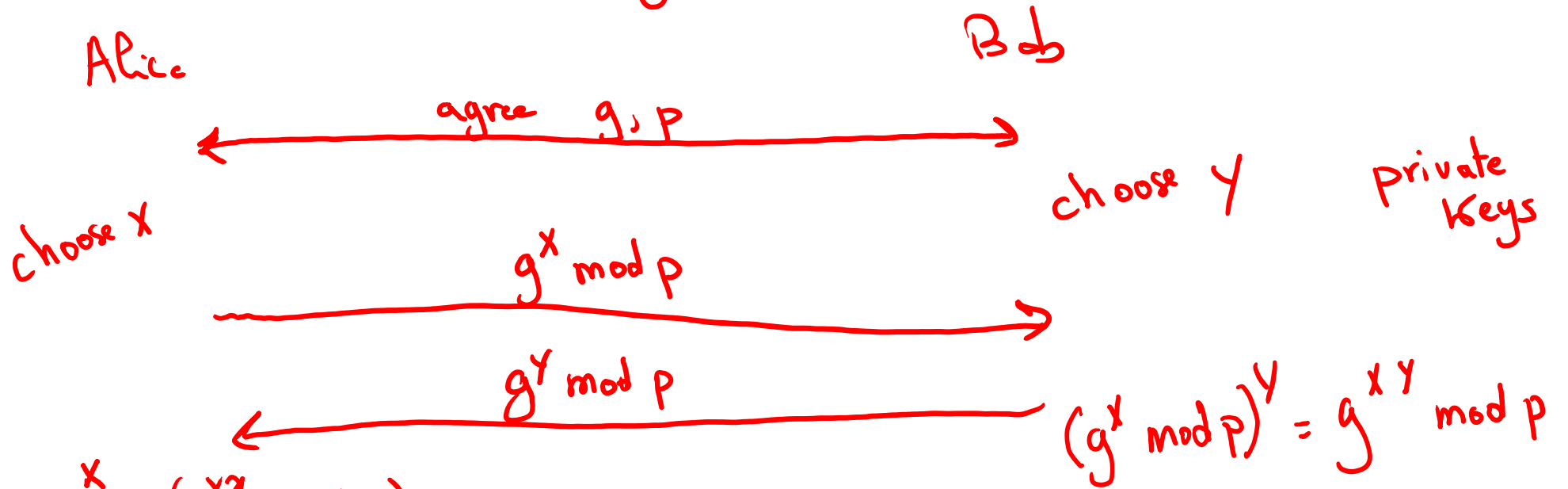
Private information

DH key exchange

$$g^x \longrightarrow x = \log_g g^x$$

integer $\longleftarrow x$

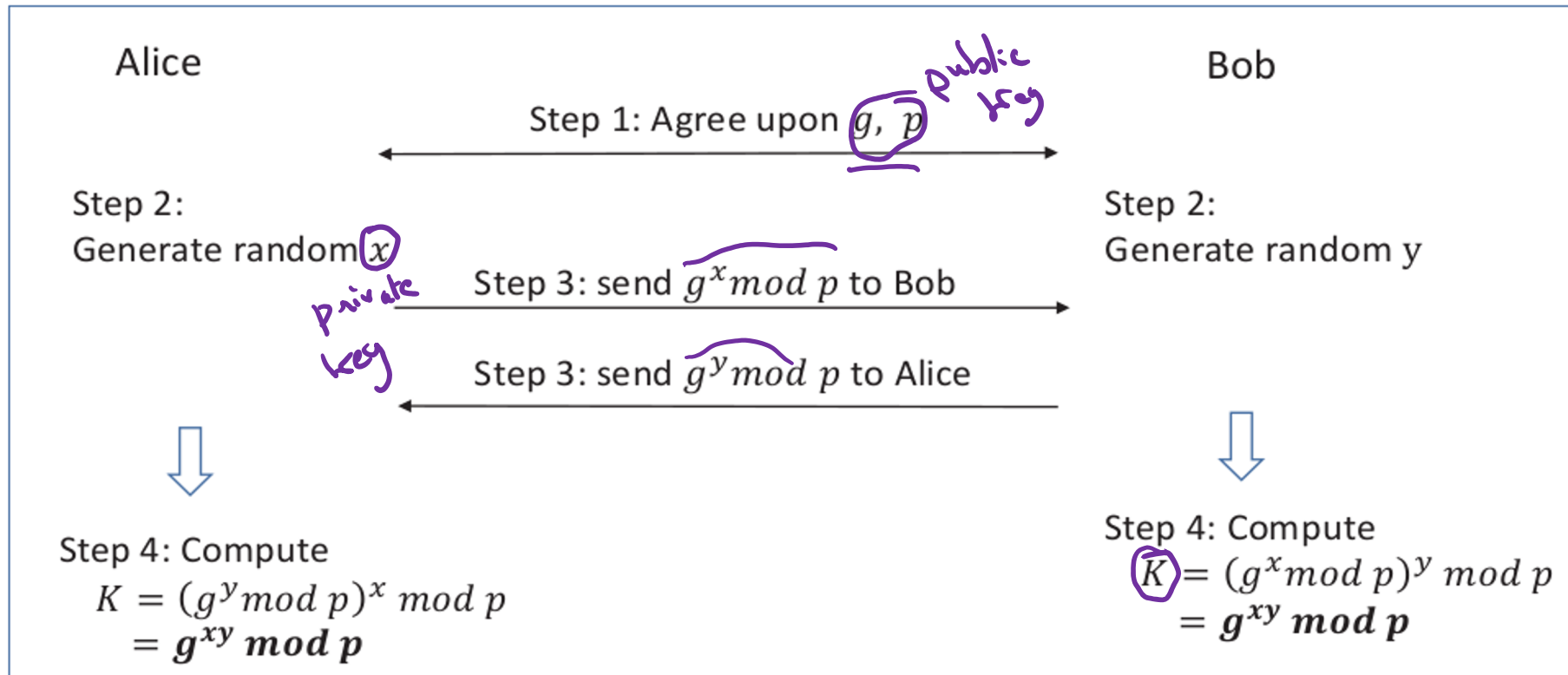
$$g^x \pmod p \longrightarrow x = ?? \quad \text{discrete logarithm}$$



$$(g^y \pmod p)^x = \underbrace{(g^{yx} \pmod p)}_K$$

Symmetric key encryption using K

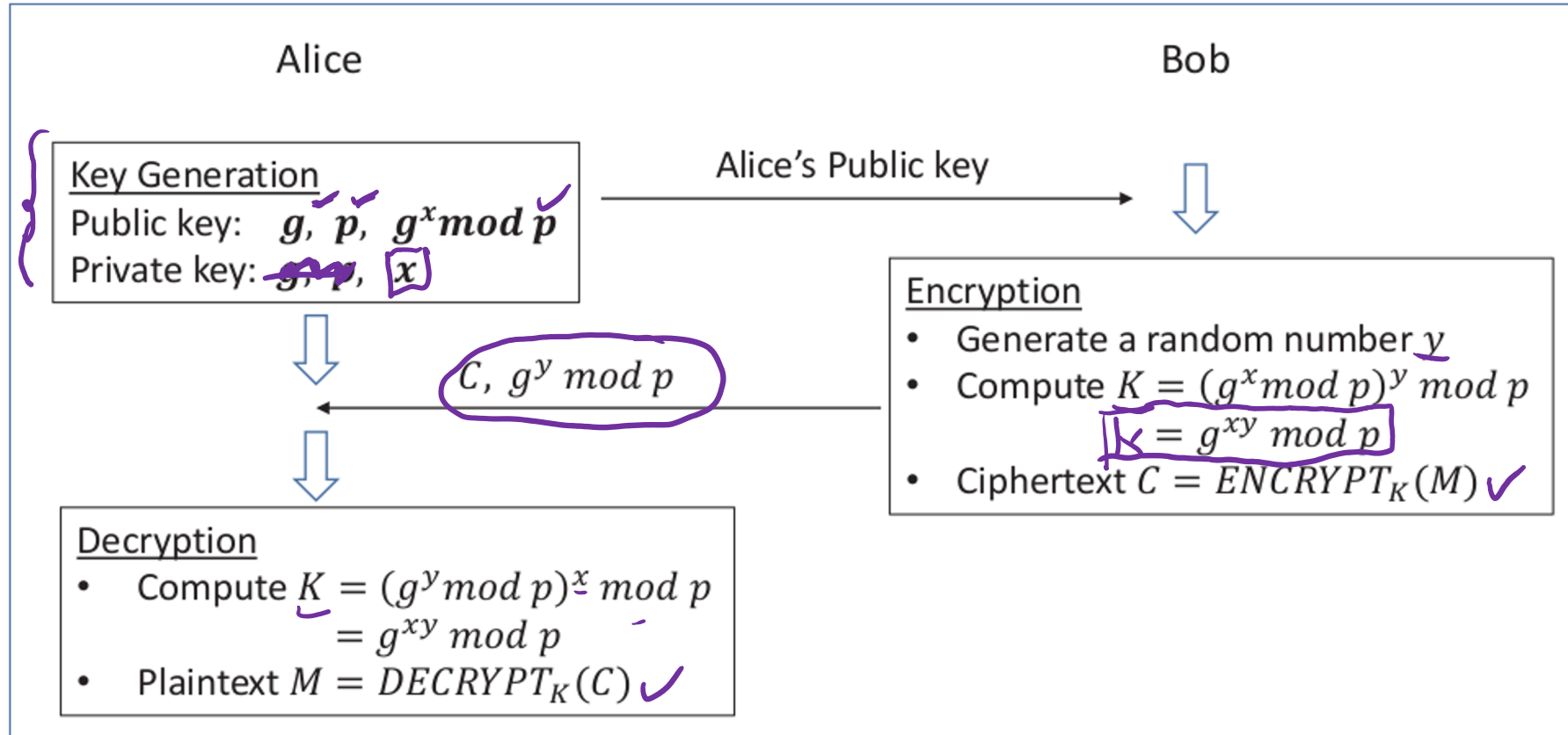
DH key exchange



DH public-key encryption

- ❑ Originally, DH allows parties to **exchange a secret key**
- ❑ What we need
 - **Public key**: known to everyone, used for encryption ✓
 - **Private key**: known to the owner, used for decryption ✓
 - **Algorithm** for encryption and decryption -

DH public-key encryption



RSA math background

$$(a + b) \bmod n = \left[(a \bmod n) + (b \bmod n) \right] \bmod n$$

$$(ab) \bmod n = \left[(a \bmod n) (b \bmod n) \right] \bmod n$$

$$a^x \bmod n = (a \bmod n)^x \bmod n$$

Euler's totient function: $\phi(n)$: # of positive integers prime to n . up to n that are relatively

a, b are relatively prime (coprimes) iff $\gcd(a, b) = 1$

e.g. 2, 3 are coprimes
3, 4 are coprimes
2, 4 are not

The RSA algorithm background

if p is prime, then $\phi(p) = p - 1$